SCEGGS Darlinghurst

2009

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics

This is a TRIAL PAPER only and does not necessarily reflect the content or format of the Higher School Certificate Examination for this subject.

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Diagrams should be drawn in pencil
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Start each question in a new booklet

Total marks – 120

- Attempt Questions 1–10
- All questions are of equal value

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Total marks – 120
Attempt Questions 1–10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks)

(a) Evaluate $\sqrt{\pi^2 - 1}$ correct to 3 significant figures. **2**

(b) Solve $|\, 4 - 2x \,| = 12$. **2**

(c) An arc of 2 cm subtends an angle of θ at the centre of a circle of radius 14 cm. Find the value of θ correct to the nearest degree. **2**

(d) Differentiate $3x^2 - \sin 2x$. **2**

(e) Simplify $\dfrac{1}{x^2 - 1} - \dfrac{1}{x + 1}$. **2**

(f) Sketch the curve $y = e^x - 1$. **2**

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) Solve $\tan^2 x = 3$ for $0 \le x \le \pi$ **2**

(b) Differentiate with respect to x:

(i) $x \tan x$ **2**

(ii) $\dfrac{\ln x}{x^2}$ **2**

(c) Find:

(i) $\displaystyle\int \frac{2x-1}{x^2 - x}\ dx$ **2**

(ii) $\displaystyle\int_0^{\pi} \sin\frac{x}{2}\ dx$ **2**

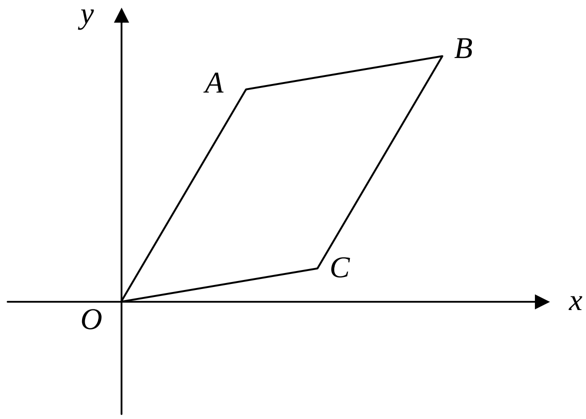
(d) Given α and β are the roots of the equation $3x^2 - 2x + 6 = 0$, find **2**

$$\alpha^2 + \beta^2$$

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a)



NOT TO SCALE

OABC is a parallelogram. Points *O*, *A* and *C* are $(0, 0)$, $(2, 6)$ and $(4, 2)$ respectively.

(i) Find the length of the interval *OC*. **1**

(ii) Find the equation of the line passing through *O* and *C* in general form. **2**

(iii) Find the midpoint of *AC*. **1**

(iv) Hence or otherwise, find the co-ordinates of *B*. **1**

(v) Find the perpendicular distance from *A* to *OC*. **2**

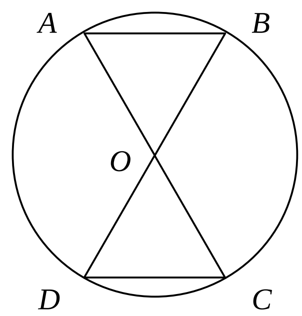
(vi) Find the area of the parallelogram *OABC*. **1**

Question 3 continues on page 5

Marks

Question 3 (continued)

(b)



NOT TO SCALE

O is the centre of the circle radius 10 cm.

(i) Prove the triangle *OAB* and *ODC* are congruent. **2**

(ii) If $\angle AOB = \frac{\pi}{5}$, find, in exact form, the area of the sector *OBC*. **2**

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

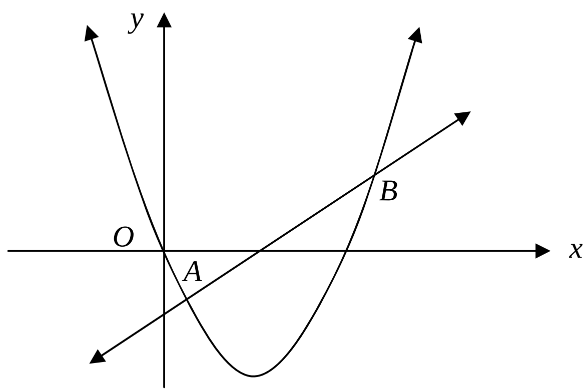
(a) (i) Express the equation of the parabola **2**

$$x^2 + 2x + 25 = 8y$$

in the form $(x-h)^2 = 4a(y-k)$.

(ii) Hence, find the focus and the equation of the directrix of the parabola. **2**

(b)



NOT TO SCALE

The curve $y = x^2 - 2x$ and the straight line $2x - y - 3 = 0$ intersect at the points A and B as shown.

(i) Find the x co-ordinates of A and B. **1**

(ii) Find the area contained between the straight line and the curve. **3**

(c) Prove that the equation of the tangent to the curve $y = \ln 2x$ at the point where $x = e$ is given by the equation **4**

$$x = e(y - \ln 2)$$

Marks

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) Find the approximation to the area bounded by the curve $y = \ln(x-2)$ between $x = 3$ and $x = 5$, correct to 2 decimal places. **3**

Use the Trapezoidal Rule and 4 subintervals.

(b) The mass *M* kg of a radioactive substance present after *t* years is given by the equation

$$M = M_0 \, e^{-kt}$$

where *k* is a positive constant.

After 50 years the substance has been reduced from 20 kg to 10 kg in mass.

(i) Show that $\dfrac{dM}{dt} = -kM$. **1**

(ii) State the value of M_0 . **1**

(iii) Find the exact value of *k*. **2**

(iv) Find the time taken for the substance to lose $\frac{4}{5}$ of its original mass. **2**

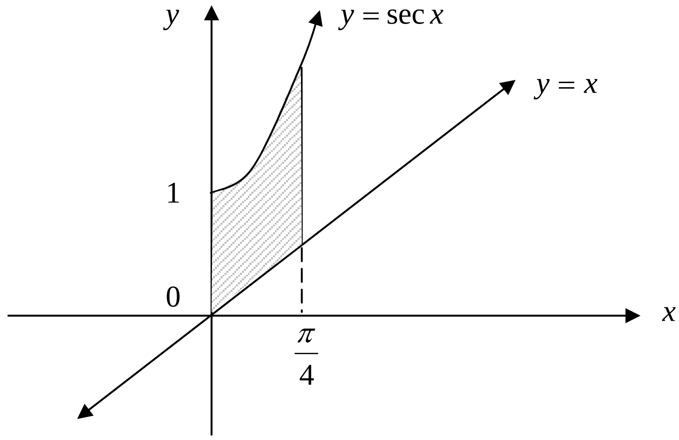
Answer to the nearest year

Question 5 continues on page 8

Marks

Question 5 (continued)

(c) **3**



NOT TO SCALE

The shaded region is bounded by the y axis, $y = x$ and the curve $y = \sec x$ from $x = 0$ to $x = \frac{\pi}{4}$.

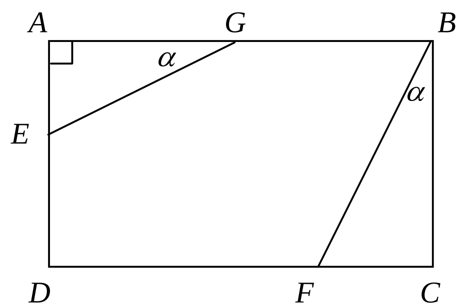
Find the volume formed when this region is rotated about the x axis. Answer in exact form.

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a) Find the condition for the equation $kx^2 - 4x + k = 0$ to have equal roots. **2**

(b)



NOT TO SCALE

$ABCD$ is a rectangle and $\angle AGE = \angle CBF = \alpha$.

Use similar triangles to prove: AG × FC = AE × BC **2**

(c) The third term of a geometric progression is 12 and the seventh term is 192. Find the first four terms of any sequence for which this is true. **3**

(d) A box contains 6 cards. Each card is labelled with a number. The numbers on the cards are 0, 1, 2, 2, 3, 3. Ruby draws the first card then a second card at random, without the first card being replaced.

(i) Find the probability that she draws a "3" followed by a "2". **1**

(ii) Find the probability that the sum of the two cards is at least 5. **2**

(iii) Find the probability that the second card withdrawn is a "2". **2**

Marks

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) The exterior angle of a regular polygon is $\frac{\pi}{10}$ radians.

(i) What is the size of each interior angle in radians? **1**

(ii) How many sides does this regular polygon have? **1**

(b) A student decides to save money over one year. In her first week she puts aside 10c. In the second week 40c, in the third week 70c, and so on with constant increases over time.

(i) What amount will she put aside in her 52nd week? (Answer in dollars.) **1**

(ii) How much has she saved altogether over the year? (Answer in dollars.) **2**

(c) (i) Prove the identity **2**

$$\frac{1}{\sin\theta + 1} - \frac{1}{\sin\theta - 1} = 2\sec^2\theta.$$

(ii) Hence find $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left(\frac{1}{\sin\theta + 1} - \frac{1}{\sin\theta - 1} \right) d\theta$ correct to 1 decimal place. **2**

(d) Solve $x^{-4} - 3x^{-2} - 4 = 0$. **3**

Marks

Question 8 (12 marks) Use a SEPARATE writing booklet.

(a) Given $y = f(x)$ is an odd function, evaluate $\int_{-a}^{a} f(x)\ dx$. **2**

Give a reason for your answer.

(b) A school basketball team has a probability of 0.75 of losing or drawing any match and a probability of 0.25 of winning any match.

(i) Find the probability of the team winning at least one of 4 consecutive matches. (Answer to 2 decimal places.) **1**

(ii) What is the least number of matches the team must play to be 95% certain of winning at least one match? **2**

(c) Jasper borrowed \$20 000 from a finance company to purchase a car. Interest on the loan is calculated quarterly at the rate of 10%p.a. and is charged immediately prior to Jasper making his quarterly repayment of \$*M*.

(i) Write an expression A_1 for the amount owing after 1 payment has been made. **1**

(ii) Show that $A_n = 20000 \times 1.025^n - 40M\left(1.025^n - 1\right)$. **2**

(iii) If the loan were to be paid out after 7 years what would the value of *M* be? **2**

(iv) If Jasper were to pay \$1282.94 per quarter in repayments, how long would it take to pay out his loan? **2**

Marks

Question 9 (12 marks) Use a SEPARATE writing booklet.

(a) Consider the curve $y = x \ln x + 2$.

(i) Find any stationary points and determine their nature. **3**

(ii) Find $\lim_{x \to 0} x \ln x + 2$. **1**

(iii) Sketch the curve showing important details. **2**

(b) Simplify $\log_b a^m \div \log_m a$ as a single expression with a logarithm of base *b*. **2**

(c) Consider the geometric series

$$2 + 2\sin^2 x + 2\sin^4 x + \ldots \quad \text{for} \quad 0 < x \le \frac{\pi}{4}.$$

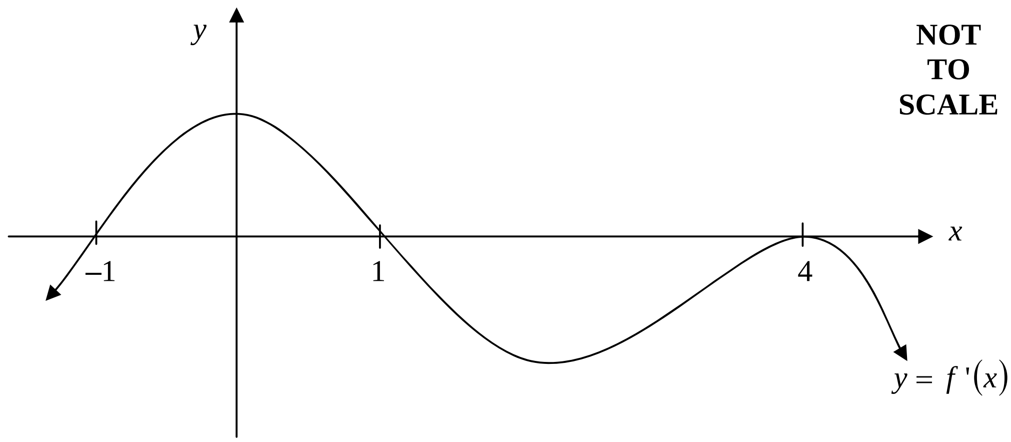
(i) Show that the limiting sum exists. **2**

(ii) Find the limiting sum if $x = \frac{\pi}{4}$. **2**

Marks

Question 10 (12 marks) Use a SEPARATE writing booklet.

(a) Copy or trace the diagram of $y = f'(x)$ given below into your answer booklet. Below this diagram, sketch $y = f(x)$ given that it passes through the points $(0, 0)$ and $(4, -2)$. Show clearly any turning points or points of inflexion. **3**



(b) ABC is a triangle with $AB = AC = x$ metres and $AB + BC + CA = 1$ metre. D is the midpoint of BC. **3**

Draw a diagram and hence prove that the perpendicular height is given by:

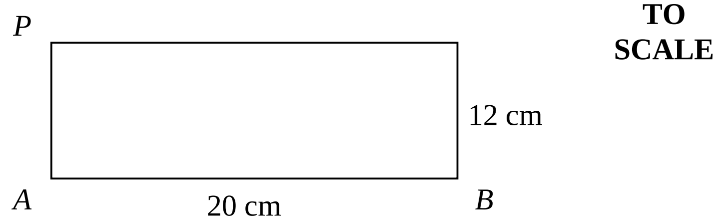
$$AD = \frac{\sqrt{4x-1}}{2} \text{ metres.}$$

Question 10 continues on page 14

Marks

Question 10 (continued)

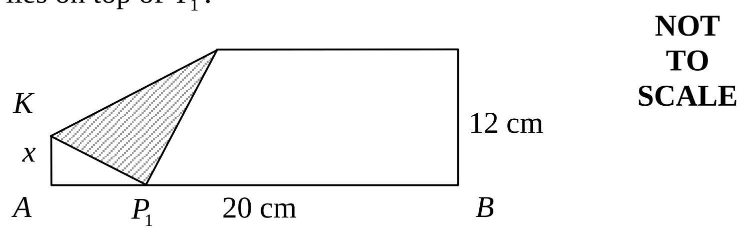
(c)



I have a rectangular sheet of paper 12 cm wide by 20 cm long.

I take the vertex labelled P and place it on the side AB.

P now lies on top of P_1.



At the bottom left of the rectangle there is a small triangle AKP_1.
Let the length of KA be x cm.

(i) Explain why KP_1 is $(12 - x)$ cm long. **1**

(ii) Show that the area of ΔAKP_1 is given by $A = x\sqrt{36 - 6x}$ **2**

(iii) Hence show that when x is one-third the length of PA the area of ΔAKP_1 is a maximum. **3**

End of Paper

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STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$c

2009 Trial Examination - Mathematics

Question 1 | Calc 2 | Comm 2 | Reas —

	Calc	Comm	Reas
Question 1	2	2	—

a) 2·98 ✓ correct answer ✓ 3 sig figs

b) $4-2x=12$ $\quad$ $-4+2x=12$

$-2x=8$ $\quad$ $2x=16$

$x=-4$ ✓ $\quad$ $x=8$ ✓

c) $l = 2\text{cm}$ $\quad$ $r = 14\text{cm}$

$l = r\theta$

$2 = 14 \times \theta$

$\theta = 0.143$ radians ✓ $\quad$ $\theta = 8^\circ$ ✓

most students arrived at $\theta = \frac{1}{7}^c$ but then made silly errors converting radians to degrees. eg 1) the case of the mysterious "π" i.e. $\frac{1}{7} \rightarrow \frac{\pi}{7}$

d) $\frac{d}{dx}\, 3x^2 - \sin 2x = 6x - 2\cos 2x$ ✓ ✓ $\quad$ **Calc 2**

must learn differentiation rules!

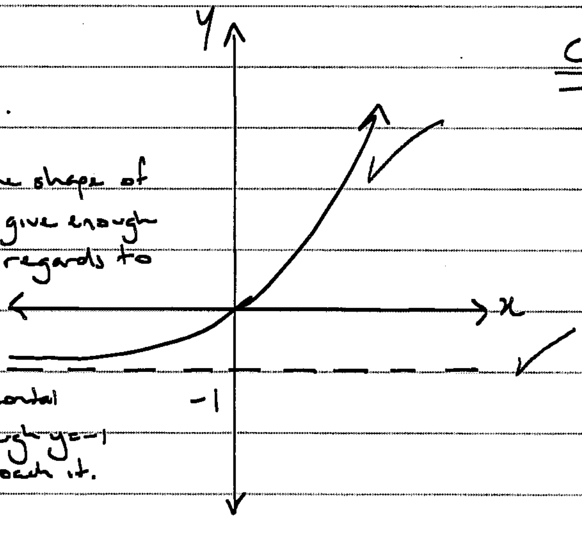
e) $\frac{1}{x^2-1} - \frac{1}{x+1} = \frac{1}{(x-1)(x+1)} - \frac{1}{(x+1)}$

$= \frac{1-(x-1)}{(x-1)(x+1)}$ ✓

$= \frac{2-x}{(x-1)(x+1)}$ ✓

Students who found the lowest common denominator were more successful than those who used $\frac{1}{(x^2-1)(x+1)}$ as a denominator.

f) $y = e^x - 1$ $\quad$ **comm 2**



Most students knew the shape of the graph but did not give enough detail, especially with regards to the asymptote at y=-1

You need to draw a horizontal dashed line passing through y=-1 and have the graph approach it.

Question 2 Calc Comm Reas

	Calc	Comm	Reas
	8	–	2

a) $\tan^2 x = 3 \quad 0 \leq x \leq \pi$

$\tan x = \pm\sqrt{3}$

(*) Many students forgot $\pm$
⇒ That is a very basic mistake that you can't afford to make.

Note because the domain is $0 \leq x \leq \pi$, this makes the answer a bit tricky (Quads 1 & 2 only)

$\tan x = \sqrt{3}$
(Quads 1 & 3)
⇒ Quad 1 only since $0 \leq x \leq \pi$

$x = \frac{\pi}{3}$ ✓

$\tan x = -\sqrt{3}$
(Quads 2 & 4)
⇒ Quad 2 only since $0 \leq x \leq \pi$

$x = \pi - \frac{\pi}{3}$
$= \frac{2\pi}{3}$ ✓

Not many students really thought about the restricted domain carefully enough. If you didn't have $\pm$ then you don't get to consider the negative case.

b) i) $u = x \quad v = \tan x$

$u' = 1 \quad v' = \sec^2 x$

(Calc 2)

Using the product rule

$y' = vu' + uv'$
$= \tan x + x\sec^2 x$

This is an easy question and was generally very well done.

ii) $u = \ln x \quad v = x^2$

$u' = \frac{1}{x} \quad v' = 2x$

(Calc 2)

Using the quotient rule

$$y' = \frac{vu' - uv'}{v^2}$$

$$= \frac{x^2 . \frac{1}{x} - 2x \ln x}{(x^2)^2} \checkmark$$

most students were able to use the quotient rule competently.

$$= \frac{x - 2x \ln x}{x^4}$$

$$= \frac{x(1 - 2\ln x)}{x^4}$$

$$= \frac{1 - 2\ln x}{x^3} \checkmark$$

Second mark awarded for correct algebraic manipulation and cancelling. This was surprisingly difficult for some.

You should automatically recognise $\int \frac{f'(x)}{f(x)} dx = \ln f(x) + C$

c) i) $\int \frac{2x - 1}{x^2 - x} dx = \ln(x^2 - x) + c$ ✓ ✓ Calc 4

must have +c for this mark

ii) $\int_0^{\pi} \sin \frac{x}{2} dx = \left[-2\cos \frac{x}{2} \right]_0^{\pi}$ ✓

Look at the standard integrals. You shouldn't get this wrong!

$$= \left[-2\cos \frac{\pi}{2} - -2\cos 0 \right]$$

$$= 2 \checkmark$$

d) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ ✓ Reas 2

$$= \frac{4}{9} - 2 \times 2$$

$$= -3\frac{5}{9} \checkmark$$

This is an easy question! You must know these off by heart.

$\alpha + \beta = -\frac{b}{a}$ $\alpha\beta = \frac{c}{a}$
$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

Question 3

	Calc	Comm	Reas
	–	–	2

a) i) $OC = \sqrt{4^2 + 2^2}$

$= 2\sqrt{5}$ ✓

ii) $M_{OC} = \frac{1}{2}$ ✓

$y - 0 = \frac{1}{2}(x - 0)$

$2y = x$

$0 = x - 2y$ ✓

General form best without fractions. This gives less problems in part (v)

iii) $M.P. (AC) = \left(\frac{2+4}{2}, \frac{6+2}{2}\right)$

$= (3, 4)$ ✓

iv) $B(6, 8)$ ✓

v) $A(2, 6)$ $\quad 0 = x - 2y$

$P.dist = \left| \frac{1\times2 - 2\times6 + 0}{\sqrt{1+4}} \right|$ ✓

$= \frac{10}{\sqrt{5}} = 2\sqrt{5}$ ✓

This was not well done. Learn formula and use it correctly.

vi) $A = 2\sqrt{5} \times 2\sqrt{5}$

$= 20\ u^2$ ✓

A basic question. Learn this technique <u>and</u> the formula.

b) i) $\angle AOB = \angle DOC$ ✓ (vertically opposite angles are equal)

$OA = OB = OC = OD$ ✓ (radii of the same circle are equal)

$\therefore \Delta OAB \equiv \Delta ODC$ (SAS) $\qquad$ Reas 2

ii) $\angle BOC = \pi - \frac{\pi}{5}$

$= \frac{4\pi}{5}$ ✓

$A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 100 \times \frac{4\pi}{5}$

$= 40\pi\ u^2$ ✓

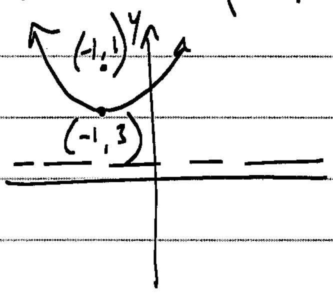
Read question! No marks if ∠BOC not found.

Question 4 — Calc 7, Comm, Reas 1

a) i) $x^2 + 2x + 1 = 8y - 25 + 1$ ✓

$(x+1)^2 = 8(y-3)$ ✓

ii) vertex $(-1, 3)$ f.l. $a = 2$



focus: $(-1, 5)$ ✓

directrix: $y = 1$ ✓

Students who drew a diagram were most successful in this question

b) i) $x^2 - 2x = 2x - 3$ — Reas 1

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 1, 3$ ✓

Well Done. No need to find the y-values, read the question

ii) Area $= \int_1^3 (2x-3) - (x^2 - 2x)\, dx$ — Calc 3

$= \int_1^3 (4x - 3 - x^2)\, dx$ ✓

$= \left[2x^2 - 3x - \frac{x^3}{3} \right]_1^3$ ✓

$= (18 - 9 - 9) - (2 + 3 - \frac{1}{3})$

$= 4\frac{1}{3}\ u^2$ ✓

some careless errors with + − but good overall

c) $y = \ln 2x$ calc 4

$y' = \frac{2}{2x} = \frac{1}{x}$ ✓

at $x = e$ $y = \ln 2e \rightarrow \ln 2 + \ln e$

$= \ln 2 + 1$

$m = \frac{1}{e}$ ✓

Eqn: $y - \ln 2e = \frac{1}{e}(x - e)$ ✓

$e[y - (\ln 2 + 1)] = x - e$ ✓

$ey - e\ln 2 - e = x - e$

$e(y - \ln 2) = x$

Most students were able to get 3 marks here. The 4th was given for correctly using log laws for the y-value

ie. $y = \ln 2e$

$= \ln 2 + \ln e$

$= \ln 2 + 1$

Question 5

Calc	Comm	Reas
3		6

a) $y = \ln(x-2)$

$h = \frac{1}{2}$ ✓

x	y	factor	Total
3	$\ln 1$	$\times 1$	
3.5	$\ln 1.5$	$\times 2$	
4	$\ln 2$	$\times 2$	
4.5	$\ln 2.5$	$\times 2$	
5	$\ln 3$	$\times 1$	

✓ table

Very poor! Learn this technique.

$\text{Area} \doteq \frac{1}{2} \div 2 \times 5.128$ Total $= 5.128$

$\doteq 1.28\ u^2$ ✓

OR by formula $A \doteq \frac{1}{2} \div 2 \left[(\ln 1 + \ln 3) + 2(\ln 1.5 + \ln 2 + \ln 2.5) \right]$ ✓ ✓

$\doteq 1.28\ u^2$ ✓

b) i) Reas 6

$M = M_0 e^{-kt}$

$\frac{dM}{dt} = -k \times M_0 e^{-kt}$ ✓

$= -kM$

Be careful differentiating exponentials.

ii) $M_0 = 20\text{ kg}$ ✓

iii) $t = 50_{yrs}$ $M = 10$, $M_0 = 20$

$10 = 20 \times e^{-k \times 50}$

$\frac{1}{2} = e^{-50k}$ ✓

$\log_e \frac{1}{2} = -50k$

$\frac{\log_e \frac{1}{2}}{-50} = k$ ✓

Sign was a problem in this question. Do not lose it!!

iv) lose $\frac{4}{5}$ means $\frac{1}{5}$ remains

This concept is important.

$$\frac{1}{5} = e^{\frac{\log_e \frac{1}{2}}{50} \times t} \quad ✓$$

$$\log_e \frac{1}{5} = \frac{t \log_e \frac{1}{2}}{50}$$

$$\frac{50 \log_e \frac{1}{5}}{\log_e \frac{1}{2}} = t$$

$$t = 116 \text{ years} \quad ✓$$

c) $V = \pi \int_0^{\frac{\pi}{4}} (\sec^2 x - x^2)\, dx$ ✓ Calc 3

$$= \pi \left[\tan x - \frac{x^3}{3} \right]_0^{\frac{\pi}{4}} \quad ✓$$

Quite well done by many students.

$$= \pi \left[\left(\tan\frac{\pi}{4} - \frac{\pi^3}{192} \right) - \left(\tan 0 - 0 \right) \right]$$

$\left(\frac{\pi}{4}\right)^3 = \frac{\pi^3}{64}$!!

$$= \pi \left(1 - \frac{\pi^3}{192} \right) u^3 \quad ✓$$

There were a few Algebra errors.

<u>OR</u> Find each volume seperately then subtract.

$$V_1 = \pi \int_0^{\frac{\pi}{4}} \sec^2 x \, dx \qquad V_2 = \pi \int_0^{\frac{\pi}{4}} x^2$$

$$= \pi \left[\tan x \right]_0^{\frac{\pi}{4}} \qquad = \pi \left[\frac{x^3}{3} \right]_0^{\frac{\pi}{4}}$$

$$= \pi \times \frac{\pi^3}{192}$$

$$= \pi \left(\tan\frac{\pi}{4} - \tan 0 \right)$$

$$= \pi$$

$$\text{Total Volume} = \pi - \frac{\pi^4}{192} \quad u^3$$

Question 6

Calc	Comm	Reas
–	–	4

a) $kx^2 - 4x + k = 0$ Reas 2

$\Delta = 0$ for equal roots

$b^2 - 4ac = 0$ ✓

$16 - 4 \times k \times k = 0$

$16 = 4k^2$

$4 = k^2$

$\pm 2 = k$ ✓ **Remember the ±**

b) $\angle AGE = \angle CBF$ (given) Reas 2

$\angle GAE = \angle BCF$ (vertices of a rectangle = 90°) ✓

$\therefore \Delta AGE \,|||\, \Delta CBF$ (equiangular)

This line must be written as you are asked to prove

↓

$\therefore \dfrac{AG}{CB} = \dfrac{AE}{CF}$ (corresponding sides of similar triangles are in proportion) ✓

$AG \times CF = AE \times CB$

many reasons were poorly written, please take the time to get this right

c) $T_3 = 12$ $T_7 = 192$

$12 = ar^2$ $192 = ar^6$

$a = \dfrac{12}{r^2}$ → $192 = \dfrac{12}{r^2} \times r^6$ ✓

$16 = r^4$

$\pm 2 = r$ ✓ **again the ±**

1 solution: 3, 6, 12, 24 ✓ either

2 solution: 3, −6, 12, −24

d) 0, 1, 2, 2, 3, 3

i) $P(3, 2) = \frac{2}{6} \times \frac{2}{5}$

$= \frac{2}{15}$ ✓

ii) sum ≥ 5

2, 3 $\left(\frac{2}{6} \times \frac{2}{5}\right) +$ ✓

3, 2 $\left(\frac{2}{6} \times \frac{2}{5}\right) +$

3, 3 $\left(\frac{2}{6} \times \frac{1}{5}\right)$

total $= \frac{1}{3}$ ✓

iii) Second card a two

1st option : 2, 2

2nd option : $\bar{2}$, 2

$P = \left(\frac{2}{6} \times \frac{1}{5}\right) + \left(\frac{4}{6} \times \frac{2}{5}\right)$ ✓ ✓

$P = \frac{1}{3}$

A common mistake in both parts ii and iii was to repeat the same outcomes too many times. Remember when you have a probability of $\frac{2}{6}$ means out of 6 there a two possibilities, do not duplicate this. ie $\frac{2}{6} + \frac{2}{6}$

Question 7	Calc	Comm	Reas
	2		5

a) Ext $\angle = \frac{\pi}{10}$

This work was not well known!

i) Int $\angle = \pi - \frac{\pi}{10}$

$= \frac{9\pi}{10}$ ✓

Learn formulae.

ii) $n = 2\pi \div \frac{\pi}{10}$

$= 20$ sides ✓

b) 10, 40, 70, ---

i) $a = 10$, $d = 30$

Well done

$T_{52} = 10 + 51 \times 30$

$= \$15.40$ ✓

ii) $S_{52} = 26 \times (0.10 + 15.40)$ ✓

Well done

$= \$403$ ✓

c) i) $LHS = \frac{1}{\sin\theta + 1} - \frac{1}{\sin\theta - 1}$

Reas 2

$= \frac{\sin\theta - 1 - \sin\theta - 1}{\sin^2\theta - 1}$ ✓

$\sin^2 + \cos^2 = 1$

$-\cos^2\theta = \sin^2\theta - 1$

$= \frac{-2}{-\cos^2\theta}$ ✓

Be really careful of the minus sign in this question

$= 2\sec^2\theta$

$= RHS$

ii) $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left(2\sec^2\theta\right) d\theta$ Calc 2

$= \left[2\tan\theta\right]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$ ✓

$= \left(2\times\sqrt{3}\right) - \left(2\times-\sqrt{3}\right)$

$= 4\sqrt{3}$ ✓

d) $x^{-4} - 3x^{-2} - 4 = 0$ Reas 3

let $u = x^{-2}$

$= \frac{1}{x^2}$

Too many students omitted the $\pm$ sign!!

$u^2 - 3u - 4 = 0$

$(u-4)(u+1) = 0$

$u = 4, \; u = -1$ ✓

$\frac{1}{x^2} = 4 \qquad \frac{1}{x^2} = -1$

$x^2 = \frac{1}{4} \qquad x^2 = -1$

↓

$x = \pm\frac{1}{2}$ no solution

✓ ✓

must have both

Question 8

	Calc	Comm	Reas
	—	2	3

a) $$\int_{-a}^{a} f(x)\,dx = 0$$

Comm 2

since $f(x)$ is an odd function it has rotational symmetry about the origin. This means that the area bounded by the curve and the x-axis from $x=-a$ to $x=0$ will be equal in magnitude to the area from $x=0$ to $x=a$ but opposite in sign hence resulting in a value of 0. ✓ ✓

Not well done! Many students thought they had to explain how to find the area rather than the value of the integral. Look for the key words underlined above

b)

Lose = 0.75 win = 0.25

Reas 3

i) $P(\text{win at least 1 of 4}) = 1 - P(\text{no wins from 4})$

$= 1 - (0.75 \times 0.75 \times 0.75 \times 0.75)$

$= 1 - 0.75^4$

$= 0.68$ ✓

ii) $P(\text{win at least 1}) = 0.95$

Not well done, learn this technique

$1 - 0.75^n = 0.95$

$0.75^n = 0.05$

$n \log 0.75 = \log 0.05$

$n = \dfrac{\log 0.05}{\log 0.75}$ ✓

$n = 10.4$

$\therefore$ they must play at least 11 games ✓

c) i) $P = 20\,000$, $r = 0.1$ p.a.
$= 0.025$ p.q.

$A_1 = 20\,000 \times 1.025 - M$ ✓

ii) $A_2 = (20\,000 \times 1.025 - M) \times 1.025 - M$
$= 20\,000 \times 1.025^2 - M(1.025 + 1)$

$A_3 = [20\,000 \times 1.025^2 - M(1.025+1)] \times 1.025 - M$
$= 20\,000 \times 1.025^3 - M(1.025^2 + 1.025 + 1)$
$\vdots$
$A_n = 20\,000 \times 1.025^n - M\underbrace{(1.025^{n-1} + 1.025^{n-2} + \dots + 1)}$ ✓

1st mark given for the expession An with the series written. 2nd mark for an expression for Sn →

GP with $a = 1$, $r = 1.025$, n
$S_n = \frac{1(1.025^n - 1)}{0.025}$
$S_n = 40(1.025^n - 1)$ ✓

$\therefore A_n = 20\,000 \times 1.025^n - 40M(1.025^n - 1)$

↙ forgotten by many students

iii) 7 years = 28 quarters
$0 = 20\,000 \times 1.025^{28} - 40M(1.025^{28} - 1)$ ✓
$40M = \frac{20000 \times 1.025^{28}}{1.025^{28} - 1}$
$M = \$1001.76$ ✓

iv) $M = \$1282.94$

$$0 = 20000 \times 1.025^n - 40 \times 1282.94(1.025^n - 1)$$

$$51317.6(1.025^n - 1) = 20\,000 \times 1.025^n$$

$$51317.6 \times 1.025^n - 51317.6 = 20\,000 \times 1.025^n$$ ✓

$$1.025^n(51317.6 - 20000) = 51317.6$$

$$1.025^n = 1.63861854$$

$$n \log 1.025 = \log 1.63861854$$

$$n = 20.00005 \text{ quarters}$$ ✓

$\therefore$ 20 quarters or 5 years

Not well done, practice these!

Question 9

Calc	Comm	Reas
4	2	4

a) $y = x \ln x + 2$ $\quad u = x \quad v = \ln x$

$u' = 1 \quad v' = \frac{1}{x}$

(Calc 4) (i & ii)

using the product rule

$y' = vu' + uv'$

$= \ln x + x \cdot \frac{1}{x}$

$= \ln x + 1$

> This part was poorly done.
> Things to notice.
> - The derivative of 2 is zero. It's gone!
> - The original question does not say $y = x \ln(x+2)$
>
> There would clearly be a bracket if that was meant to be the function.

$y' = \ln x + 1$

stat pts when $y' = 0$

$\ln x = -1$ ←

> Many students got stuck at this line. Change it to base e and then use the definition to find x.
>
> $\log_a x = N$
>
> $a^N = x$

$\log_e x = -1$

By definition

$e^{-1} = x$

$\therefore x = \frac{1}{e}$ ✓

$\doteq 0.37$

$y = \frac{1}{e} \ln e^{-1} + 2$

$= -\frac{1}{e} + 2$ ✓

$= 2 - \frac{1}{e}$

$\doteq 1.6$

Test nature

Method 1: Using $y'' = \frac{1}{x}$	Method 2: using $y' = \ln x + 1$
at $x = \frac{1}{e}$ $y'' = e$ $y'' > 0$ concave up ∪	see table below

x	$\frac{1}{5}$	$\frac{1}{e}$	1
y'	-0.609	0	2
	$-$	0	$+$
	\	—	/

i. $\therefore$ Minimum Turning point at $\left(\frac{1}{e}, -\frac{1}{e}+2\right) \doteq (0.4, 1.6)$ ✓

ii) $\lim_{x \to 0} x \ln x + 2$

$\lim_{x \to 0} x = 0$

$\lim_{x \to 0} \ln x = \infty$

Use logic to think about each part separately

$\therefore \lim_{x \to 0} x \ln x = 0$

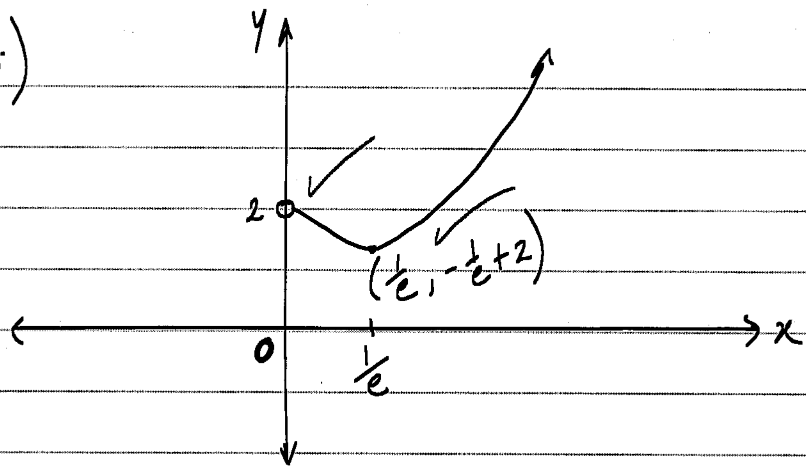
$\therefore \lim_{x \to 0} x \ln x + 2 = 2$ ✓

This part was very challenging for most students. You have to show your findings to the graph you draw in part iii). Make sure to link your ideas.

Before drawing the graph, here are some considerations

- One minumum TP at $\left(\frac{1}{e}, 2-\frac{1}{e}\right)$
- No possible P.O.I since $y'' = \frac{1}{x} \neq 0$ for any x. This means no change in concavity.
- The domain of $\ln x$ is $x > 0$ so your graph cannot have any part drawn left of the y axis.

iii)



Comm 2

The graph has an open circle at $(0,2)$ because the limit of the curve is 2 as $x \to 0$ (from part ii)

A number of students thought this meant a horizontal asymptote $y=2$ as $x \to \infty$ which is incorrect.

As $x \to \infty$, the curve $\to \infty$.

b) $\log_b a^m \div \log_m a$

(Reas 2)

$= m\log_b a \div \dfrac{\log_b a}{\log_b m}$ ✓

$= m\log_b a \times \dfrac{\log_b m}{\log_b a}$

$= m\log_b m$ ✓

If you go further

$= \log_b m^m$ (not $\log_b m^2$)

First mark awarded for change of base. The new base must be clearly shown.

This question highlights the need to learn the log laws properly.

$\dfrac{\log a}{\log b} \neq \log a - \log b$

$\log\left(\frac{a}{b}\right) = \log a - \log b$

c) i) $2 + 2\sin^2 x + 2\sin^4 x + \ldots$

$$r = \frac{T_2}{T_1} = \frac{T_3}{T_2} = \sin^2 x$$

One mark awarded for $r = \sin^2 x$ and $-1 < r < 1$

for $0 < x \leq \frac{\pi}{4}$

$\sin^2 0 < \sin^2 x \leq \sin^2 \frac{\pi}{4}$

$0 < \sin^2 x \leq \frac{1}{2}$ ✓

Second mark awarded for correct inequality $0 < \sin^2 x \leq \frac{1}{2}$

since $-1 < r < 1$ then the limiting sum exists. ✓

If you start with $-1 < r < 1$

$-1 < \sin^2 x < 1$

you have to go into more detail and think logically.

- $\sin^2 x$ is always positive $\therefore \sin^2 x \geq 0$
  but $r = 0$ has no meaning $\therefore \sin^2 x > 0$
- for $0 < x < \frac{\pi}{4}$ $\sin^2 \frac{\pi}{4} = \left(\frac{1}{\sqrt{2}}\right)^2$
  $= \frac{1}{2}$ is the highest value for $\sin^2 x$

$\therefore 0 < \sin^2 x \leq \frac{1}{2}$

---

ii) $S_\infty = \frac{a}{1-r}$

$= \frac{2}{1 - \sin^2 \frac{\pi}{4}}$ ✓

$= 2 \div \frac{1}{2}$

$= 4$ ✓

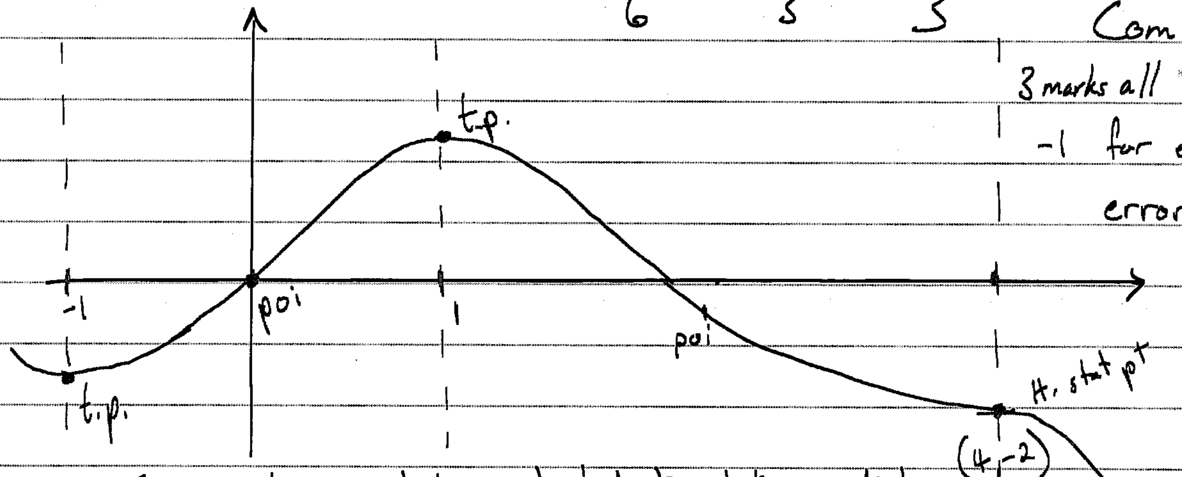
(*) You can do this part (ii) even if you couldn't do part (i)

Mark awarded for correct formula and substitution of $\frac{\pi}{4}$

Question 10

Calc Comm Reas
6 3 3

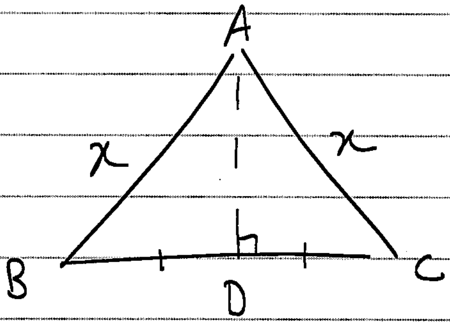
a)



Com 3
3 marks all 5
-1 for each error

This was done poorly by most students and this needs to be revised as the work for this was done some time ago! It is also important to label points. i.e. turning points or horizontal points of inflexion.

b)



$BD = \frac{1-2x}{2}$ ✓ Reas 3

ΔABC is isosceles, since $AB = AC$ since the bisector of the base of an isosceles triangle is perpendicular then $AD \perp BC$

$$AD^2 = AB^2 - BD^2$$

$$AD^2 = x^2 - \left(\frac{1-2x}{2}\right)^2$$

$$= x^2 - \frac{1+4x-4x^2}{4}$$

$$= x^2 - \frac{1}{4} + x - x^2 \quad ✓$$

$$AD^2 = x - \frac{1}{4}$$

$$AD = \sqrt{\frac{4x-1}{4}}$$

$$AD = \frac{\sqrt{4x-1}}{2} \quad ✓$$

c) i) $PA = 12$ cm

$PK = P_1K$
$PK = 12 - x$ ✓
$\therefore P_1K = 12 - x$

Reas 3

Make sure when answering 'show' questions that you actually explain how you arrive at your answer. This is especially important when it is an obvious question.

ii) $AP_1^2 = (12-x)^2 - x^2$

$= 144 - 24x + x^2 - x^2$

$AP_1^2 = 144 - 24x$

$AP_1 = 2\sqrt{36-6x}$ ✓

$A = \frac{1}{2} \times AK \times AP_1$ ✓

$= \frac{1}{2} \times x \times 2\sqrt{36-6x}$

$= x\sqrt{36-6x}$

iii) a max when $A' = 0$

Calc 3

$u = x \qquad v = (36-6x)^{\frac{1}{2}}$

$u' = 1 \qquad v' = -3(36-6x)^{-\frac{1}{2}}$

$A' = \sqrt{36-6x} - \dfrac{3x}{\sqrt{36-6x}}$ ✓

With a difficult differentiation it is important to take your time so as not to make algebraic errors.

if $A' = 0$

$\dfrac{3x}{\sqrt{36-6x}} = \sqrt{36-6x}$

$3x = 36 - 6x$

$9x = 36$

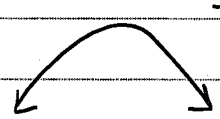
$x = 4$ ✓

Remember it is far more efficient to use the first derivative test for a maximum, with a difficult differentiation rather than the second derivative

test nature when $x = 4$

x	3	4	5
A'	$\sqrt{18} - \frac{9}{\sqrt{18}}$	0	$\sqrt{11} - \frac{25}{\sqrt{11}}$
	+		−

✓



$\therefore$ a maximum

since 4 is $\frac{1}{3}$ of 12

max when x is $\frac{1}{3}$ of PA